CHAPTER 9 CHANNELS

APPENDIX A

Hydraulic Design Equations for Open Channel Flow

OFFICE OF STRUCTURES

SEPTEMBER 2009

CHAPTER 9 APPENDIX A

Hydraulic Design Equations for Open Channel Flow

Introduction

The Equations presented in this Appendix are, for the most part, the basic open channel flow equations covered in standard text books. During the re-edit of Chapter 9 Channels, a determination was made to retain these equations for the use of the Manual user, but to relocate them to this Appendix. Please contact the Office of Structures, H&H Structures Division if you have comments or questions relating to the use of these equations.

9.4 Open Channel Flow

9.4.1 General

Design analysis of both natural and artificial channels proceeds according to the basic principles of open channel flow (Reference 23). The basic principles of fluid mechanics -- continuity, momentum, and energy -- can be applied to open channel flow with the additional complication that the position of the free surface is usually one of the unknown variables. The determination of this unknown is one of the principal problems of open channel flow analysis and it depends on quantification of the flow resistance. Natural channels display a much wider range of roughness values than artificial channels.

9.4.2 Definitions

Specific Energy

Specific energy E is defined as the energy level relative to the channel bottom. If the channel is not too steep (slope less than 10 percent) and the streamlines are nearly straight and parallel (so that the hydrostatic assumption holds), the specific energy E becomes the sum of the depth and velocity head:

$$\mathbf{E = y + \alpha (V^2/2g)} \qquad \textbf{(9.1)}$$

where:

y = depth, ft

α = kinetic energy correction coefficient

V = mean velocity, ft/s

g = gravitational acceleration, 32.2 ft/s^2

The kinetic energy correction coefficient is taken to have a value of one for turbulent flow in prismatic channels but may be significantly different than one in natural channels.

Kinetic Energy Correction Coefficient

As the velocity distribution in a river varies from a maximum at the design portion of the channel to essentially zero along the banks, the average velocity head, computed as $(Q/A)^2/2g$ for the stream at a section, does not give a true measure of the kinetic energy of the flow. A weighted average value of the kinetic energy is obtained by multiplying the average velocity head, above, by a kinetic energy coefficient, α, defined as:

$$\boldsymbol{\alpha = [\Sigma(qv^2)/(QV^2]} \qquad \textbf{(9.2)}$$

Where:

v = average velocity in subsection, ft/s

q = discharge in same subsection, cfs

Q = total discharge in river, cfs

V = average velocity in river at section or Q/A, ft/s

9.4.2 Definitions (continued)

Total Energy Head

The total energy head is the specific energy head plus the elevation of the channel bottom with respect to some datum. The locus of the energy head from one cross section to the next defines the energy grade line. See Figure 9-1 for a plot of the specific energy diagram.

Steady and Unsteady Flow

A steady flow is one in which the discharge passing a given cross-section is constant with respect to time. The maintenance of steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

Uniform Flow and Non-uniform Flow

A non-uniform flow is one in which the velocity and depth vary in the direction of motion, while they remain constant in uniform flow. Uniform flow can only occur in a prismatic channel, which is a channel of constant cross section, roughness, and slope in the flow direction; however, non-uniform flow can occur either in a prismatic channel or in a natural channel with variable properties.

Gradually-varied and Rapidly-Varied

A non-uniform flow in which the depth and velocity change gradually enough in the flow direction that vertical accelerations can be neglected, is referred to as a gradually-varied flow; otherwise, it is considered to be rapidly-varied.

Froude Number

The Froude number is an important dimensionless parameter in open channel flow. It represents the ratio of inertia forces to gravity forces and is defined by:

$$\mathbf{F = V/(gd)^{.5}} \qquad \mathbf{(9.3)}$$

Where:

- V = mean velocity = Q/A, ft/s
- g = acceleration of gravity, ft/s^2
- d = hydraulic depth = A/T, ft
- A = cross-sectional area of flow, ft^2
- T = channel top width at the water surface, ft

This expression for Froude number applies to any single section channel of non-rectangular shape.

9.4.2Definitions (continued)

Critical Flow

The variation of specific energy with depth at a constant discharge shows a minimum in the specific energy at a depth called critical depth at which the Froude number has a value of one. Critical depth is also the depth of maximum discharge when the specific energy is held constant. These relationships are illustrated in Figure 9-1.

Subcritical Flow

Depths greater than critical occur in subcritical flow and the Froude number is less than one. In this state of flow, small water surface disturbances can travel both upstream and downstream, and the control is always located downstream.

Supercritical Flow

Depths less than critical depth occur in supercritical flow and the Froude number is greater than one. Small water surface disturbances are always swept downstream in supercritical flow, and the location of the flow control is always upstream.

9.4 Open Channel Flow (continued)

Hydraulic Jump

A hydraulic jump occurs as an abrupt transition from supercritical to subcritical flow in the flow direction. There are significant changes in depth and velocity in the jump, and energy is dissipated. For this reason, the hydraulic jump is often employed to dissipate energy and control erosion at highway drainage structures.

9.4.3 Flow Classification

The classification of open channel flow can be summarized as follows.

Steady Flow

1. Uniform Flow

2. Non-uniform Flow

a. Gradually Varied Flow

b. Rapidly Varied Flow

Unsteady Flow

1. Unsteady Uniform Flow (rare)

2. Unsteady Non-uniform Flow

a. Gradually Varied Unsteady Flow

b. Rapidly Varied Unsteady Flow

The steady uniform flow case and the steady non-uniform flow case are the most fundamental types of flow used in highway engineering hydraulic models like HEC-2 and WSPRO for purposes of developing water surface profiles. However, the gradually varied unsteady flow case is used for purposes of evaluating tidal flow.

9.4 Open Channel Flow (continued)

9.4.4 Equations

The following equations are those most commonly used to analyze open channel flow. The use of these equations in analyzing open channel hydraulics is discussed in Section 9.5.

Manning's Equation

For a given channel geometry, slope, and roughness, and a specified value of discharge Q, a unique value of depth occurs in steady uniform flow. It is called the normal depth. The normal depth is used to design artificial channels in steady, uniform flow and is computed from Manning's equation:

$$\mathbf{Q = (1.49/n)AR^{2/3}S^{1/2}} \qquad \textbf{(9.4)}$$

Where:

Q = discharge, cfs

n = Manning's roughness coefficient

A = cross-sectional area of flow, ft^2

R = hydraulic radius = A/P, ft

P = wetted perimeter, ft

S = channel slope, ft/ft

9.4.4 Equations (continued)

The selection of Manning's n is generally based on observation; however, considerable experience and judgment is necessary to arrive at an appropriate n value for a given site. A study by the Corps of Engineers (Reference 39) has shown that errors in the selection of Manning n values will normally result in greater errors in the elevation of flood water surface profiles than errors in any of the other variable used in the preparation of hydraulic studies of streams and their flood plains. **Accordingly, the Engineer is expected to carefully apply the procedures developed by the U.S. Geological Survey as presented in Reference 22 to determine n values for channels and their flood plains. (See also Chapter 3, Appendix A)**

Reference 5, also prepared by the U.S. Geological Survey, and is recommended for use as a guide for checking whether the values obtained from the procedures in Reference 22 are reasonable.

The most accurate estimates of Manning's n values can be obtained at locations where high water marks are available and can be correlated with a known discharge.

9.4 Open Channel Flow (continued)

The following guidance is offered with regard to the selection of appropriate Manning's n values:

- For major flood flows, the bed of sand channels can be expected to assume the plain bed form of the upper regime.
- Particular care should be exercised in the estimation of n values for wooded flood plains. There is an apparent tendency for Engineers to underestimate flood plain n values, and consequently to overestimate the amount of overbank flow. Errors of this type result in unrealistic values of live bed contraction scour, as explained in Chapter 10.

Additional discussion of Manning's n values is contained in Section 9.5.2.1.

If the normal depth computed from Manning's equation is greater than critical depth, the slope is classified as a mild slope, while on a steep slope, the normal depth is less than critical depth. Thus, uniform flow is subcritical on a mild slope and supercritical on a steep slope.

9.4.4 Equations

In channel analysis, it is often convenient to group the channel properties in a single term called the channel conveyance K:

$$\mathbf{K = (1.49/n)AR^{2/3}} \qquad \mathbf{(9.5)}$$

and then Manning's Equation can be written as:

$$\mathbf{Q = KS^{1/2}} \qquad \mathbf{(9.6)}$$

The conveyance represents the carrying capacity of a stream cross-section based upon its geometry and roughness characteristics alone and is independent of the streambed slope.

The concept of channel conveyance is useful when computing the distribution of overbank flood flows in the stream cross section and the flow distribution through the opening in a proposed stream crossing. Appendix 9C presents an illustration of how conveyance can be used to determine the distribution of flood flow within the channel and on the flood plain.

9.4 Open Channel Flow (continued)

Continuity Equation

The continuity equation is the statement of conservation of mass in fluid mechanics. For the special case of steady flow of an incompressible fluid, it assumes the simple form:

$$\mathbf{Q = A_1V_1 = A_2V_2} \quad \mathbf{(9.7)}$$

Where:

Q = discharge, cfs

A = flow cross-sectional area, ft^2

V = mean cross-sectional velocity, ft/s (which is perpendicular to the cross section)

The subscripts 1 and 2 refer to successive cross sections along the flow path. The continuity equation can be used together with Manning's Equation to obtain the steady uniform flow velocity as:

$$\mathbf{V = Q/A = (1.49/n)R^{2/3}S^{1/2}} \quad \mathbf{(9.8)}$$

Energy Equation

The energy equation expresses conservation of energy in open channel flow expressed as energy per unit weight of fluid which has dimensions of length and is therefore called energy head. The energy head is composed of potential energy head (elevation head), pressure head, and kinetic energy head (velocity head). These energy heads are scalar quantities which give the total energy head at any cross section when added. Written between an upstream open channel cross section designated 1 and a downstream cross section designated 2, the energy equation is:

$$\mathbf{h_1 + \alpha_1(V_1^2/2g) = h_2 + \alpha_2(V_2^2/2g) + h_L} \quad \mathbf{(9.9)}$$

Where: h_1 and h_2 are the upstream and downstream stages, respectively, ft

α = kinetic energy correction coefficient

V = mean velocity, ft/s

h_L = head loss due to local cross-sectional changes (minor loss) as well as boundary resistance, ft

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y, i.e. h=z+y. The energy equation states that the total energy head at an upstream cross section is equal to the energy head at a downstream section plus the intervening energy head loss. The energy equation can only be applied between two cross sections at which the streamlines are nearly straight and parallel so that vertical accelerations can be neglected.

9.5.3 Step-backwater Models

The computation of water surface profiles by HEC-RAS is based on the standard step method in which the stream reach of interest is divided into a number of sub reaches by cross sections spaced such that the flow is gradually-varied in each sub reach. The energy equation is then solved in a step-wise fashion for the stage at one cross section based on the stage at the previous cross section.

The method requires definition of the geometry and roughness of each cross section as discussed in Section 9.5.1. Manning's n values can vary both horizontally across the section as well as vertically. Expansion and contraction head loss coefficients, variable main channel and overbank flow lengths, and the method of averaging the slope of the energy grade line can all be specified.

Step-backwater analysis is useful for determining unrestricted water surface profiles where a highway crossing is planned, and for analyzing how far upstream the water surface elevations are affected by a culvert or bridge. Because the calculations involved in this analysis are tedious and repetitive, it is recommended that a computer program such as the Corps of Engineers HEC-RAS be used.

The HEC-RAS Version 4.0 and HEC-2 programs developed by the Corps of Engineers are widely used for calculating water surface profiles for steady gradually varied flow in natural and man-made channels. SHA recommends HEC-RAS for most locations except where previous flood studies for FEMA or other flood control agencies have been made. In this case, it may be easier to use the existing HEC-2 models in order to compare existing and proposed water surface profiles. Both subcritical and supercritical flow profiles can be calculated by these programs. The effects of bridges, culverts, weirs, and structures in the floodplain may be also considered in the computations. These programs are also designed for application in flood plain management and flood insurance studies.

9.5.3 Step-Backwater Models (continued)

To amplify on the methodology, the energy equation is repeated from Section 9.4.4:

$$\mathbf{h_1 + \alpha_1(V_1^2/2g) = h_2 + \alpha_2(V_2^2/2g) + h_L} \qquad \mathbf{(9.10)}$$

Where:

h_1 and h_2 are the upstream and downstream stages, respectively, ft

α = kinetic energy correction coefficient

V = mean velocity, ft/s

h_L = head loss due to local cross-sectional changes (minor loss) as well as boundary resistance, ft

The stage h is the sum of the elevation head z at the channel bottom and the pressure head, or depth of flow y, i.e., h = z+y. The energy equation is solved between successive stream reaches with nearly uniform roughness, slope, and cross-sectional properties.

The total head loss is calculated from:

$$\mathbf{h_L = K_m[(\alpha_1 V_1^2/2g) - (\alpha_2 V_2^2/2g)] + S_e L} \qquad \mathbf{(9.11)}$$

Where:

K_m = the minor loss coefficient

S_e = the mean slope of the energy grade line evaluated from Manning's equation and a selected averaging technique (Shearman, 1990 and HEC-2), ft/ft

These equations are solved numerically in a step-by-step procedure called the Standard Step Method from one cross section to the next.

The default values of the minor loss coefficient K_m are zero and 0.1 for contractions and 0.5 and 0.3 for expansions in WSPRO and HEC-2, respectively. For HEC-RAS, the default coefficients for typical bridge sections are 0.3 for contractions and 0.5 for expansions.

9.5.4 Profile Computation

Water surface profile computation requires a beginning value of elevation or depth (boundary condition) and proceeds upstream for subcritical flow and downstream for supercritical flow. In the case of supercritical flow, critical depth is often the boundary condition at the control section, but in subcritical flow, uniform flow and normal depth may be the boundary condition. The starting depth in this case can either be found by the single-section method (slope-area method) or by computing the water surface profile upstream to the desired location for several starting depths and the same discharge. These profiles should converge toward the desired normal depth at the control section to establish one point on the stage-discharge relation. If the several profiles do not converge, then the stream reach may need to be extended downstream, or a shorter cross-section interval should be used, or the range of starting water-surface elevations should be adjusted. In any case, a plot of the convergence profiles can be a very useful tool in such an analysis.

Given a long enough stream reach, the water surface profile computed by step-backwater will converge to normal depth at some point upstream for subcritical flow. Establishment of the upstream and downstream boundaries of the stream reach is required to define limits of data collection and subsequent analysis. Calculations must begin sufficiently far downstream to assure accurate results at the structure site, and continued a sufficient distance upstream to accurately determine the impact of the structure on upstream water surface profiles